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# Compositional Neural Certificates for Networked Dynamical Systems

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**Website**

## Problem

Developing stable controllers for large-scale networked dynamical systems.

**Scalability** **Network Generalizability** **Robustness**

## Scalability

**Theorem 1** Suppose each subsystem has a decentralized controller $u_i = \pi_i(x_i)$ and a continuously differentiable function $V_i(x_i)$. Suppose (1) For each $i$, there exists $\mathcal{K}_\infty$ functions $\underline{\alpha}_i, \bar{\alpha}_i$ such that $\underline{\alpha}_i \mathrm{dist}\left(x_i, \mathcal{X}_i^{\mathrm{goal}}\right) \le V_i(x_i) \le \bar{\alpha}_i \mathrm{dist}\left(x_i, \mathcal{X}_i^{\mathrm{goal}}\right)$; (2) For each $i$, there exists $\alpha_i > 0$ and class-$\mathcal{K}$ functions $\chi_{ij}, j \in \mathcal{N}_i$ satisfying $\chi_{ij}(a) < a, \forall a > 0$, such that $\forall x_i, x_{\mathcal{N}_i}$,

$$V_i(x_i) \ge \max_{j \in \mathcal{N}_i} \chi_{ij}\left(V_j(x_j)\right)$$
$$\Rightarrow [\nabla V_i(x_i)]^\top f_i\left(x_i, x_{\mathcal{N}_i}, \pi_i(x_i)\right) \le -\alpha_i V_i(x_i).$$

Then, the closed-loop system under controller $\pi_1, \dots, \pi_n$ is globally asymptotically stable around $\mathcal{X}^{\mathrm{goal}}$. Such functions $V_i(x_i)$ are called **ISS Lyapunov functions**.

## Network Generalizability

**Theorem 2** Consider a networked dynamical system $\mathcal{N}, \mathcal{N}_i, f_i$ with ISS Lyapunov functions $V_i$. Suppose there is another system $\tilde{\mathcal{N}}, \tilde{\mathcal{N}}_i, \tilde{f}_i$. Suppose for each $j \in \tilde{\mathcal{N}}$, there exists a one-to-one map $\tau_j: \{j\} \cup \tilde{\mathcal{N}}_j \to \mathcal{N}$ such that $\tau_j(\tilde{\mathcal{N}}_j) = \mathcal{N}_{\tau_j(j)}$, and $\tilde{f}_j = f_{\tau_j(j)}$. Further, suppose $\forall j, j' \in \tilde{\mathcal{N}}, \forall l \in \tilde{\mathcal{N}}_j \cap \tilde{\mathcal{N}}_{j'}$, we have $V_{\tau_j(l)} = V_{\tau_{j'}}(l)$. Then $\tilde{\pi}_j = \pi_{\tau_j(j)}$ is a stabilizing controller for the new system with compositional certificate $\tilde{V}_j = V_{\tau_j}(j)$.

## Robustness

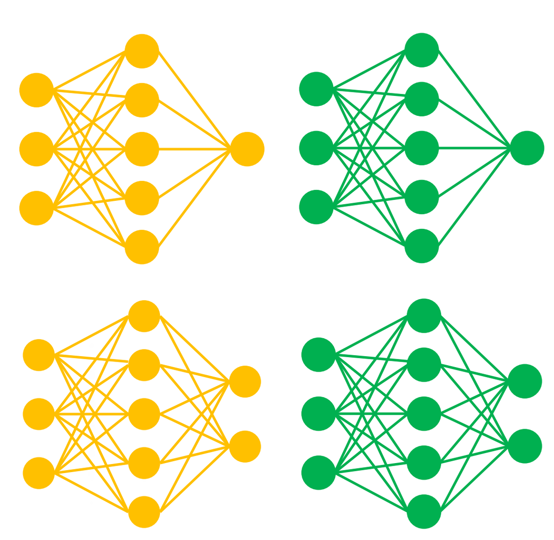
**Theorem 3** Given a networked dynamical system with control-affine dynamics with bounded parametric uncertainty $\beta \in \mathcal{B}$, where $\mathcal{B}$ is the convex hull of parameters $\beta_1, \beta_2, \dots, \beta_{n_\beta}$. If there exists ISS Lyapunov functions $V_i$ satisfying the conditions for each $\beta_j, j \in \{1,2, \dots n_\beta\}$, the dynamics $h_i$ and $g_j$ are affine with respect to $\beta$, then the closed-loop system is globally asymptotically stable with any $\beta \in \mathcal{B}$.

## Learning Neural ISS Lyapunov functions (NeurISS)

**Sharable ISS-Lfs and Controllers**

ISS-LF

Controller



**Loss**

How to encode the "imply" condition?

$$A_i = \left[V_i(x_i) \ge \max_{j \in \mathcal{N}_i} \chi_{ij}\left(V_j(x_j)\right)\right], B_i = \left[[\nabla V_i(x_i)]^\top f_i\left(x_i, x_{\mathcal{N}_i}, \pi_i(x_i)\right) \le -\alpha_i V_i(x_i)\right]$$

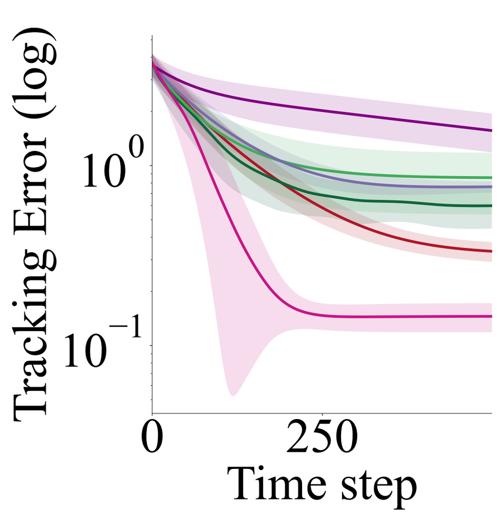
The "imply" condition: $A_i \Rightarrow B_i$ is the same as $\neg A_i \vee B_i$, or $\max\{\neg A_i, B_i\}$.

$$\mathcal{L}_{A_i} = \mathrm{ReLU}\left(V_i(x_i) - \max_{j \in \mathcal{N}_i} \chi_i(V_j(x_j) + \epsilon_A\right),$$
$$\mathcal{L}_{B_i} = \mathrm{ReLU}\left([\nabla V_i(x_i)]^\top f_i\left(x_i, x_{\mathcal{N}_i}, \pi_i(x_i)\right) + \alpha_i V_i(x_i) + \epsilon_B\right),$$
$$\mathcal{L} = \sum_{i=1}^n \left[\sum_{x_i^{\mathrm{goal}}} \left|V_i\left(x_i^{\mathrm{goal}}\right)\right| + \mu_{A_i}\mathcal{L}_{A_i} + \mu_{B_i}\mathcal{L}_{B_i} + \mu_{\mathrm{ctrl}}\|\pi_i(x_i) - u_i^{\mathrm{nom}}\|^2\right].$$

## Experiments

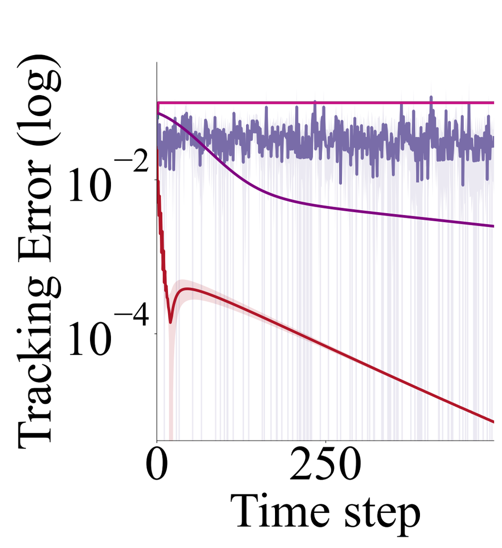
**Baselines:** Centralized: PPO (Proximal Policy Optimization), LYPPO (PPO with Lyapunov critic), NCLF (Neural centralized CLF); Decentralized: MAPPO (Multi-agent PPO), LQR (Linear Quadratic Regulator)

### Networked Microgrids



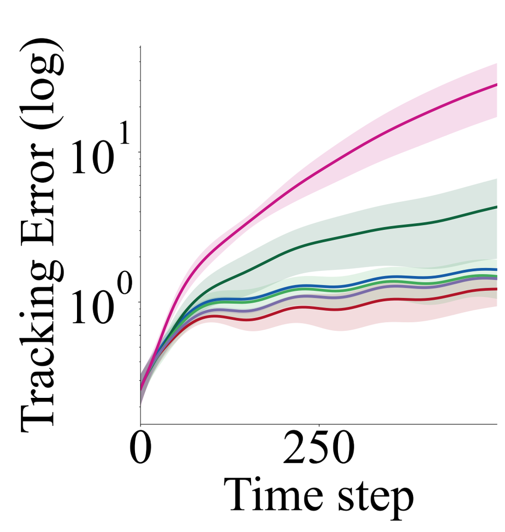

The IEEE 123-node test feeder divided into 5 subsystems. We want to stabilize the frequency and the voltage.

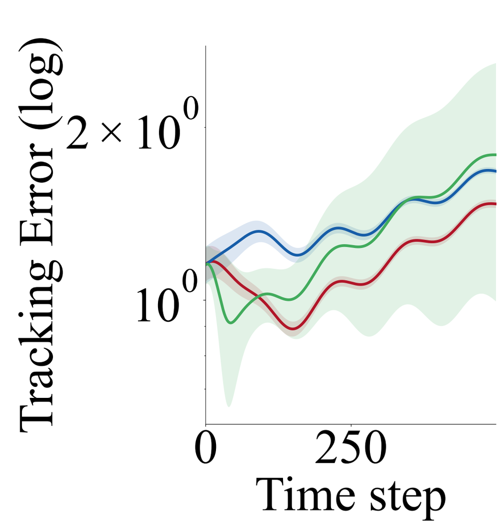
### Power Grid (8 buses)




Voltage control with 8-bus power grid

### Truck Platoon

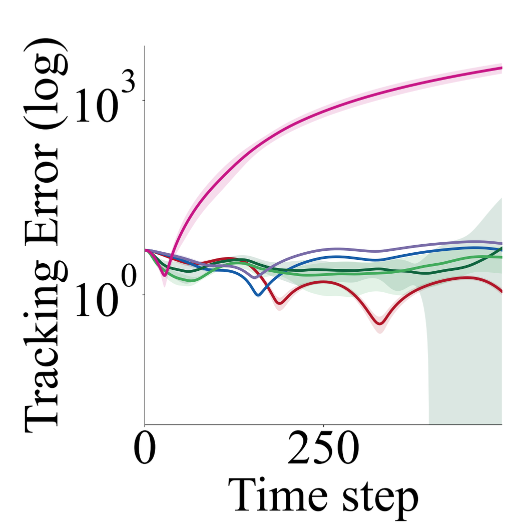


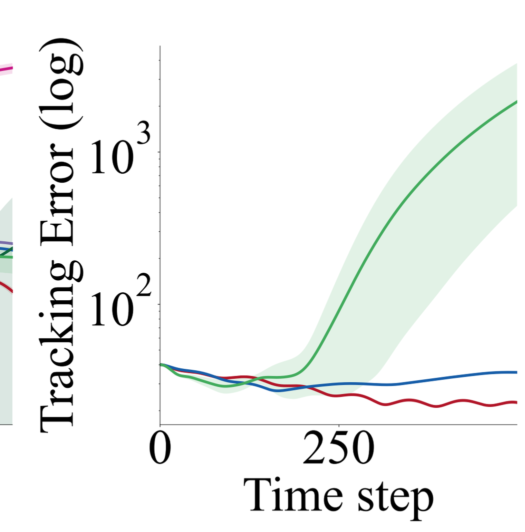




5 trucks 100 trucks

Keep the platoon formation and track some unknown speed profile. We train on 5 trucks and test on both 5 and 100 trucks.

### Drone Formation Control




4 drones 100 drones

Keep the drone formation and track some unknown speed profile. We train on 4 drones and test on both 4 and 100 drones.

NeurISS PPO MAPPO LYPPO NCLF LQR (Nominal) Droop (Nominal)